Name of College — S. S. College, J. Bad

Subject — Mathematics

Topic — Infinite Series (Real Analysis)

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B.Sc Part-II (Hons)

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Infinite Series :→

If $<u_n>$ be a sequence of real No.
then the Expression
$u_1 + u_2 + u_3 + \dots + u_n + \dots$ is defined.
as infinite series
it is usually denoted by $\sum_{n=1}^{\infty} u_n$
or simply $\sum u_n$

$S_n = u_1 + u_2 + u_3 + \dots + u_n$

S_n = sum upto First n Terms.

Ex:→ $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + \dots + n \dots$

$S_n = 1 + 2 + 3 + \dots + n$

Positive Term Series :→ the series $\sum u_n$ is called positive term series if $u_n > 0 \; \forall n \in N$

Alternating series :→ An infinite series of Type
$u_1 - u_2 + u_3 - u_4 + \dots$
Where the Terms are alternately positive and Negative is defined as an alternating Series.

Convergent series →

A series $\sum u_n$ is said to be Convergent if the seem of its first n Terms Can not Numerically Exceed a Finite quantity however Large n may be.

i.e Σu_n is convergent if

$\lim_{n\to\infty} S_n$ = finite and unique.

<u>Divergent Series :→</u>

Σu_n is divergent if

$$\lim_{n\to\infty} S_n = +\infty \text{ or } = -\infty$$

<u>Oscillating series :→</u>

Oscillating series are of two types

(a) A series Σu_n is said to oscillate finitely

if $\lim_{n\to\infty} S_n$ = finite but not unique

(b) A series Σu_n is said to oscillate infinitely if $\lim_{n\to\infty} S_n = +\infty$

or $\lim_{n\to\infty} S_n = -\infty$.

<u>Remarks</u> :→ (i) The convergency or divergency of a series is not affected by altering, adding or neglecting a finite no of its terms

(ii) The convergency or divergency

of a series is not affected by the Multiplication of all the Terms of the Series by a Fixed a Non-Zero No

(iii) An infinite Series in which all the Terms are of the same sign is divergent if each Term is Greater than some Finite quantity however small.

Solved Example.

1. Show that the series

$$1+3+5+7+\cdots\cdots \text{ is divergent}$$

Solution :→ Given series

$$1+3+5+7+\cdots\cdots\cdots.$$

Here $S_n = \frac{n}{2}[2\times1+(n-1)\times2]$

$$= \frac{n}{2}[2+2n-2]$$

$$= n^2$$

$\therefore \lim_{n\to\infty} S_n = \lim_{n\to\infty} n^2 = \infty$

$\therefore$ the Given Series is divergent.

Ex:→ Discuss the Convergency of the Series

$$\frac{1}{1\cdot3}+\frac{1}{2\cdot4}+\frac{1}{3\cdot5}+\frac{1}{4\cdot6}+\cdots\cdots$$

Solution →

Given Series is

$$\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \frac{1}{4\cdot 6} + \cdots$$

$$\therefore u_n = \frac{1}{n(n+2)}$$

$$= \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right]$$

putting $n = 1, 2, 3, \ldots n$

$$u_1 = \frac{1}{2}\left[1 - \frac{1}{3}\right]$$

$$u_2 = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$u_3 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$u_4 = \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right]$$

$$\cdots$$

$$u_{n-2} = \frac{1}{2}\left[\frac{1}{n-2} - \frac{1}{n}\right]$$

$$u_{n-1} = \frac{1}{2}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$u_n = \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right]$$

On Adding we get

$$S_n = \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$\Rightarrow S_n = \frac{1}{2}\left[\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$\therefore \lim_{n\to\infty} S_n = \frac{1}{2}\lim_{n\to\infty}\left[\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{1}{2}\lim_{n\to\infty}\left[\frac{3}{2} - \frac{1}{n(1+1/n)} - \frac{1}{n(1+2/n)}\right]$$

$$= \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$ which is finite and unique.

Hence the given series is convergent.

Test the convergent of the series $\sum_{n=0}^{\infty} (-1)^n$.

Soln

Here $\sum u_n = \sum_{n=0}^{\infty} (-1)^n$

$S_n = (-1)^0 + (-1)^1 + (-1)^2 + (-1)^3 + \cdots$ to n terms

$= 1 - 1 + 1 - 1 + \cdots$ to n terms

$= 1$ when n is odd

$= 0$ when n is zero

$\therefore \lim_{n\to\infty} S_n = 1$ or 0 i.e finite but not unique.

Hence, $\sum u_n$ is finitely oscillating series